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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

DEM5038 – ENGINEERING MATHEMATICS 3

(Diploma in Electronics Engineering)

8 MARCH 2019 3.00 p.m. – 5.00 p.m. (2 Hours)

INSTRUCTIONS TO STUDENT

- 1. This question paper consists of 5 pages excluding the cover page.
- 2. Answer ALL questions. All necessary working steps must be shown.
- 3. Write all your answers in the answer booklet provided.

Please answer <u>ALL</u> questions and show the necessary working. Total mark is equal to 100.

Question 1 (25 marks)

a) Find the solution to the homogeneous differential equation:

$$y'' + 25y = 0$$
 $y(0) = 0$; $y(\frac{\pi}{10}) = 1$ [9 marks]

b) Find the solution to the non-homogeneous differential equation:

$$y''-2y'=e^{2x}$$
 $y(0)=0$; $y'(0)=1$ [16 marks]

Question 2 (25 marks)

a) Given one period of a periodic function as $f(x) = \begin{cases} -1, & -3 < x < 0 \\ 1, & 0 < x < 3 \end{cases}$

Find the Fourier series for the above function.

[16 marks]

b) Find the Half-range Sine series for $f(x) = \begin{cases} x + \pi, & -\pi < x < 0 \\ x - \pi, & 0 < x < \pi \end{cases}$; $f(x + 2\pi) = f(x)$. [9 marks]

Continued...

Question 3 (25 marks)

- Determine the Laplace transform of the following functions. a)
 - i. $3e^{4t} te^{-2t} + 5$

[2 marks]

ii. $\int_{0}^{t} 3\cos 2x dx$

[2 marks]

iii. $f(t) = \begin{cases} 2, & 0 < t < 3 \\ 0, & t \ge 3 \end{cases}$

[4 marks]

- b) Find the inverse Laplace transform for the following functions.
 - i. $\frac{1}{s+2} + \frac{2}{s^2+4}$

[2 marks]

ii. $\frac{4}{(s+3)^2-4}$

[2 marks]

Find the solution to the following differential equation by using Laplace c) transform method.

 $y''-2y'-3y = 3e^{2t}$ y(0) = 0, y'(0) = 1

[13 marks]

Continued...

Question 4 (25 marks)

- a) According to Chemical Engineering Progress in 1990, 30% of all pipework failures in chemical plants are caused by operator error. Using the binomial distribution function, find the probability that in a random sample of 20 pipework failures,
 - i. at least 10 are due to operator error.

[2 marks]

ii. not more than 4 are due to operator error.

[2 marks]

- b) The resistance of resistors is normally distributed with mean 150Ω and standard deviation of 5Ω . What percentage of the resistors will have resistance between 148Ω and 152Ω ? [2 marks]
- c) An average IQ score of school students in a city is 100. The IQ scores are normally distributed. In a recent survey, a random sample of 30 students IQ scores in a school have a mean score of 112 with a standard deviation of 15. Test at 5% significance level if the school students have higher IQ scores?

 [7 marks]

d) A study of the amount of rainfall and the quantity of air pollution removed produced the following data:

Table 1

Daily Rainfall x (0.01cm)	Particulate Removed $y (\mu g/m^3)$
4.3	126
4.5	121
5.9	116
5.6	118
6.1	114
5.2	118
3.8	132
2.1	141
7.5	108

- i. Find the estimate regression line of y on x, $\hat{y} = a + bx$.
- [10 marks]
- ii. Estimate the amount of particulate removed when the daily rainfall is x = 4.8 units. [2 marks]

End of Page.

Appendix

Quadratic formula

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Annihilator for special functions

Annihilator	
D''	$1, x, x^2, x^3, x^4, \dots, x^{n-1}$
$(D-a)^n$	$e^{ax}, xe^{ax}, x^2e^{ax}, x^3e^{ax}, x^4e^{ax}, \dots, x^{n-1}e^{ax}$
$(D^2-2aD+(a^2+b^2))^n$	$e^{ax}\cos bx, xe^{ax}\cos bx, x^2e^{ax}\cos bx, x^3e^{ax}\cos bx, \dots, x^{n-1}e^{ax}\cos bx$
	and
	$e^{ax} \sin bx$, $xe^{ax} \sin bx$, $x^2 e^{ax} \sin bx$, $x^3 e^{ax} \sin bx$,, $x^{n-1} e^{ax} \sin bx$

Fourier Series

$F(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$ $a_0 = \frac{1}{2L} \int_{-L}^{L} f(x) dx$ $a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi}{L} x dx$ $b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi}{L} x dx$

Integration Formula

$$\int \sin ax dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax dx = \frac{1}{a}\sin ax$$

$$\int x \sin ax dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x \cos ax dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2 \sin ax dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2 \cos ax dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

Linear Regression and Correlation

$$\hat{y} = a + bx$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$b = \frac{SS_{xy}}{SS_{xx}}$$

$$a = \overline{y} - b\overline{x} = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Transformation of some functions

f(t)	$L\{f(t)\}$
1	$\frac{1}{s}$, s>0
t	$\frac{1}{s^2}$
t^n	$n!$ $n = 1, 2, 3, \dots$
e ^{at}	$\frac{1}{s-a} , s>a$
t e ^{at}	$\frac{1}{\left(s-a\right)^2}$
<i>y'</i>	sY(s)-y(0)
y''	$s^2Y(s) - sy(0) - y'(0)$

f(t)	$L\{f(t)\}$
sin ωt	$\frac{\omega}{s^2 + \omega^2}$
cos ωt	$\frac{s}{s^2 + \omega^2}$
sinh ωt	$\frac{\omega}{s^2-\omega^2}$
cosh ωt	$\frac{s}{s^2-\omega^2}$
(1)u(t-a)	$\frac{e^{-sa}}{s}$
(t-a)u(t-a)	$\frac{e^{-sa}}{s^2}$

Second Shift Theorem

$$L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}\$$

$$L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}\$$

Binomial Probabilities

$$\binom{n}{x} p^x q^{n-x}$$

Binomial Formulae when using Cambridge Statistical Table

Key Formulas (if $p \le 0.5$)

1.
$$P(X = r) = B(r) - B(r - 1)$$

Key Formulas (if
$$p > 0.5$$
)

1. $P(X = r) = P(Y = n - r) = B(n - r) - B(n - r - 1)$

2. $P(X \ge r) = P(Y \le n - r) = B(n - r)$

2.
$$P(X > r) = 1 - R(r-1)$$

2.
$$P(X \ge r) = 1 - B(r - 1)$$

3. $P(X > r) = 1 - B(r)$
4. $P(X \ge r) = P(Y \le n - r) = P(x \le n - r)$
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$$3. P(X > r) = 1 - B(r)$$

3.
$$P(X \le r) = P(Y \ge n - r) = 1 - B(n - r - 1)$$

4.
$$P(a \le X \le b) = B(b) - B(a-1)$$

4.
$$P(a \le X \le b) = B(b) - B(a-1)$$
 4. $P(a \le X \le b) = P(n-b \le Y \le n-a)$

$$5. P(X \le r) = B(r)$$

$$= B(n-a) - B(n-b-1)$$

Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

Hypothesis Testing

$$S_x = \frac{S}{\sqrt{n}}$$

$$S_x = \frac{s}{\sqrt{n}} \qquad Z = \frac{\overline{x} - \mu}{S_x}$$